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RF & Microwave Solutions

RF & Microwave Engineering 101

Presentation for meetings 16-17 of 20:

Introduction to digital wireless communications

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Content for meetings 16 and 17:

Introduction to Digital Wireless Communications (continued)

16.1 Distinct signal distortion mechanisms that can be measured by a VSA

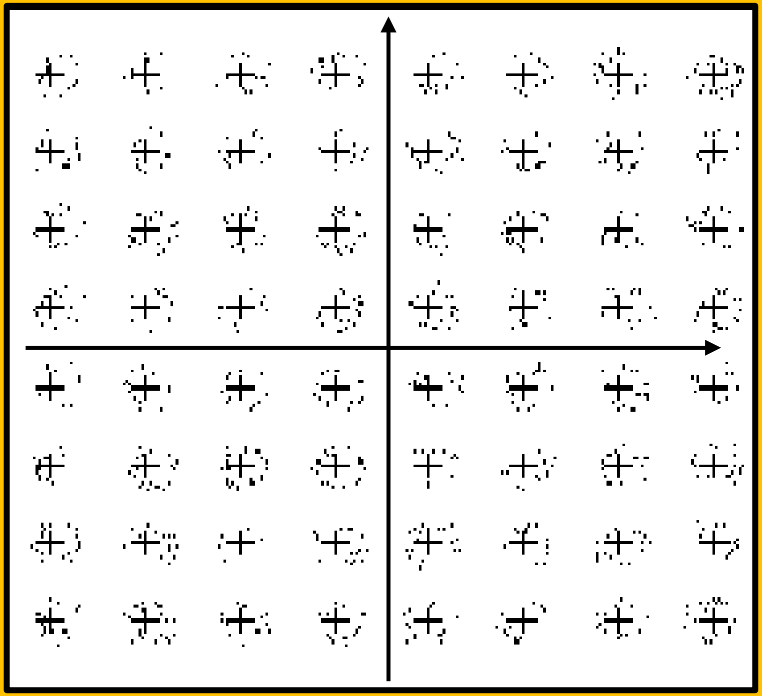
16.2 The Error vector magnitude (EVM) metric

16.3 Baseband representation and SISO system BER/SNR performance

16.1 Distinct constellation deformations

For single-carrier constellations measured in VSA

Distortion 1: Poor SNR (Constellation with strong additive noise):



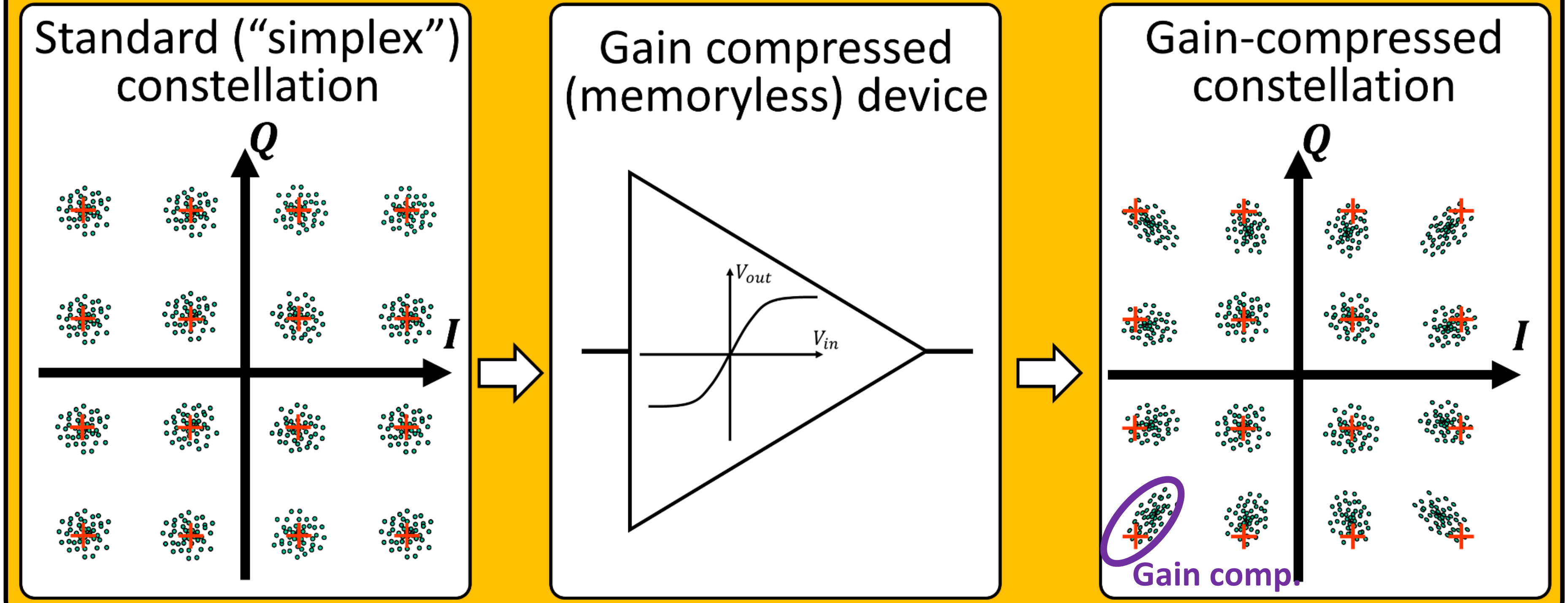
VSA measurement of a noisy (poor SNR) constellation

Voltage noise with zero-mean which is added to the standard constellation, will have a new mean value: The transmitted symbol's location. In other words, additive noise samples will appear in the VSA as "symmetric circular clouds" of vectors (points in the I/Q plane) around each standard constellation point (symbol). The lower the SNR, the wider the spread (RMS radius) of the noise clouds.

☒ **Not a systematic error**

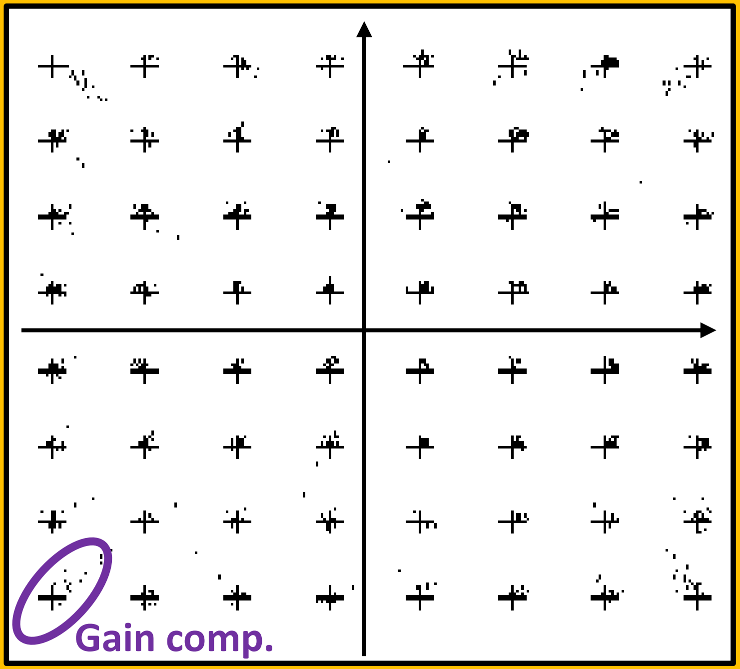
Distortion 2: AM-AM Gain compression effects:

Constellation passing through a Gain-compressed (memoryless AM-AM distortion) device:



Distortion 2: Gain compression (AM-AM), continued:

In case the constellation is inserted into an AM-AM gain-compressed device, the constellation's symbols will be "pulled towards the origin" due to the gain compression. The stronger (more far from the origin) symbols will experience this effect more strongly. In the extreme case of passing through a "hard limiter", the constellation will become circular (keeping phase information only) by this distortion.



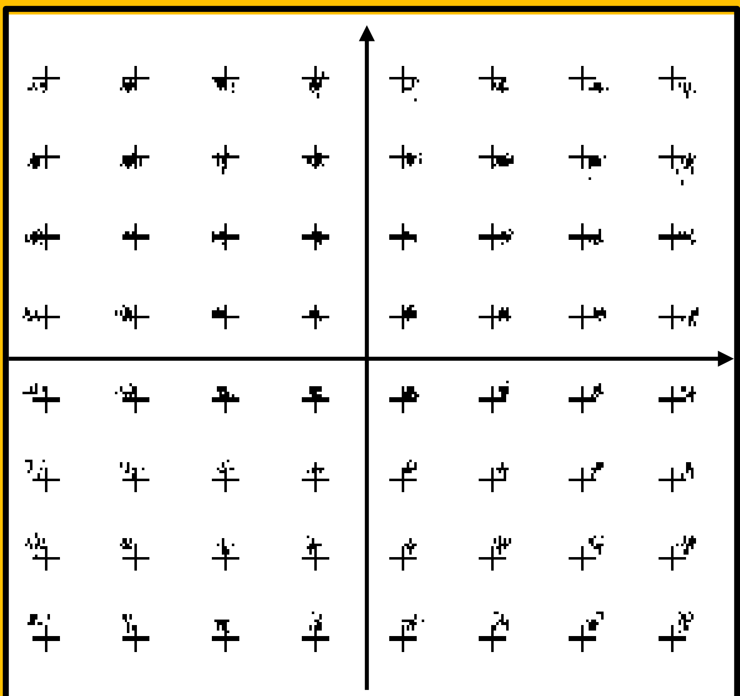
VSA measurement of AM-AM Gain-compressed constellation

☑ **Systematic error**

Distortion 3: Gain-Imbalance between the I and Q hardware channels:

In case the gain if the I and Q channels (hardware) does not exactly match, the constellation will suffer from an I/Q gain imbalance distortion.

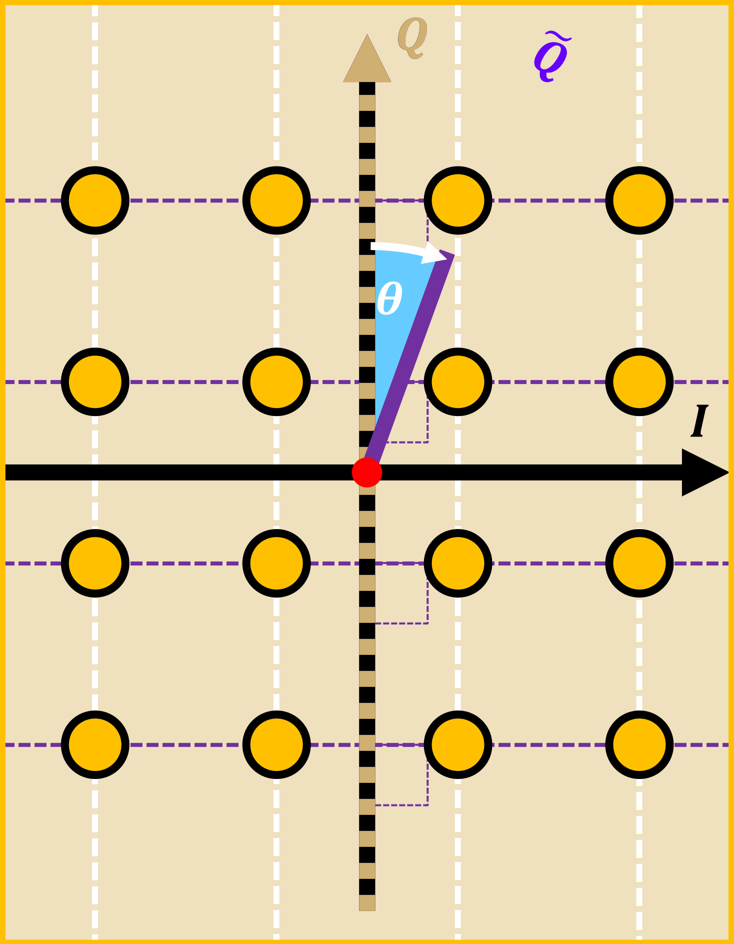
An x[dB] gain imbalance will stretch (amplify) one axis over the other. The resulting constellation will appear stretched on one axis and compressed on the other.



VSA measurement of an I/Q Gain-imbalanced constellation

☑ **Systematic error**

Distortion 4: I/Q phase imbalance



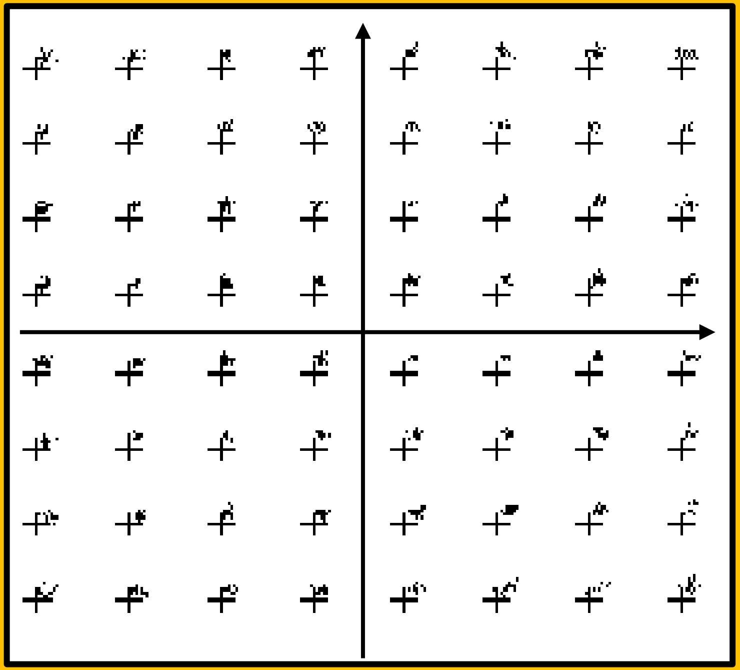
In case the phases of the I and Q carriers are not exactly aligned to be 90° apart, the constellation will suffer from an I/Q phase imbalance distortion.

A phase imbalance θ, will rotate one axis (from its standard position), while keeping its symbol projections (coordinates) at their original values.

☑ **Systematic error**

Distortion 5: DC offset (carrier leakage)

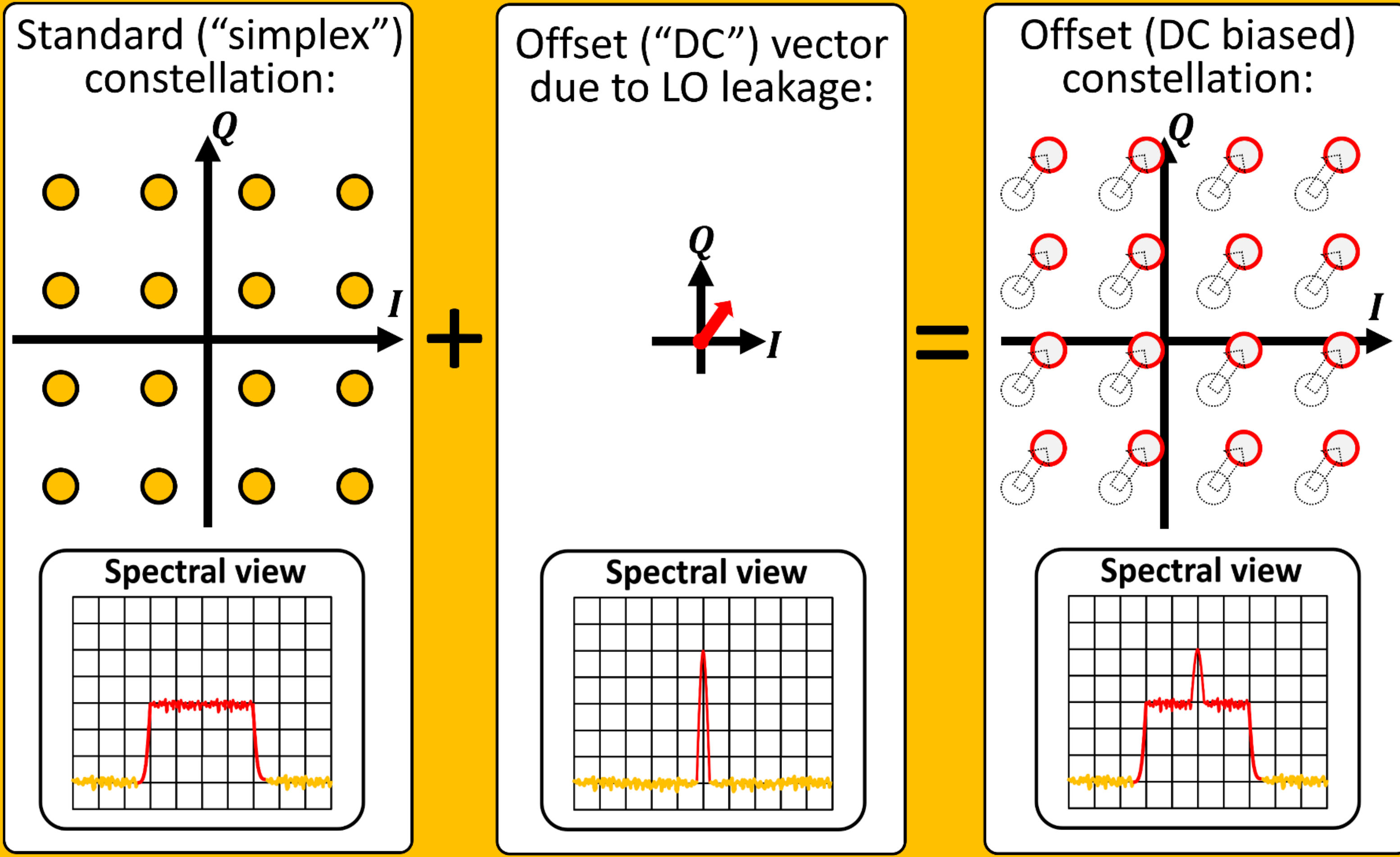
Practical LO (carrier) leakage due to the finite isolation between the LO and RF ports of the transmitter's I/Q mixers, will appear as a superimposed carrier wave that will be transmitter alongside the ideal (simplex) constellation. Since the leaking carrier is by definition phase coherent with the I and Q carriers, it will be represented by a fixed "offset vector" in the signal space, yielding an offset (non-simplex) biased constellation.



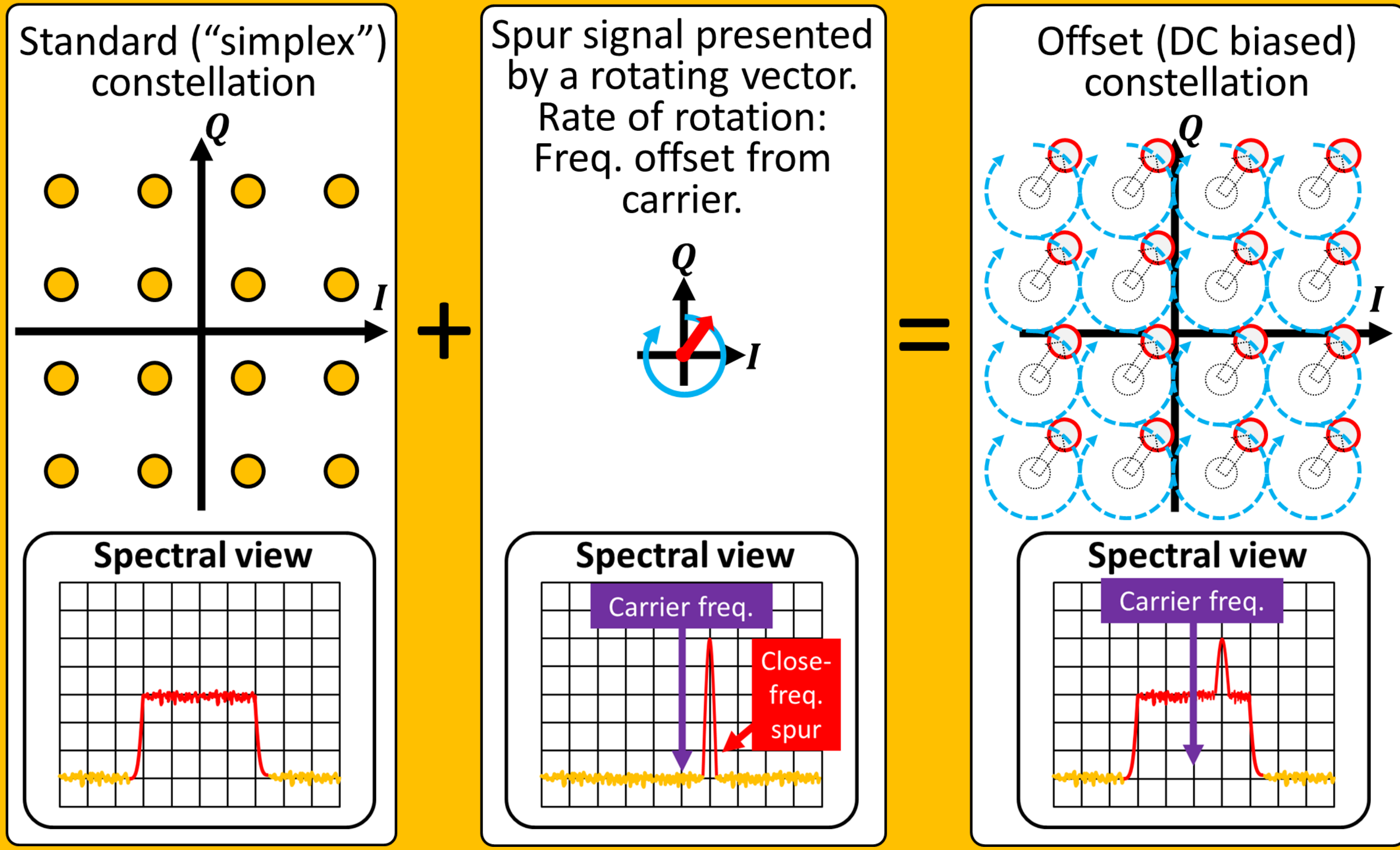
VSA measurement of DC-offset constellation

Systematic error

Distortion 5: DC offset (carrier leakage), continued:

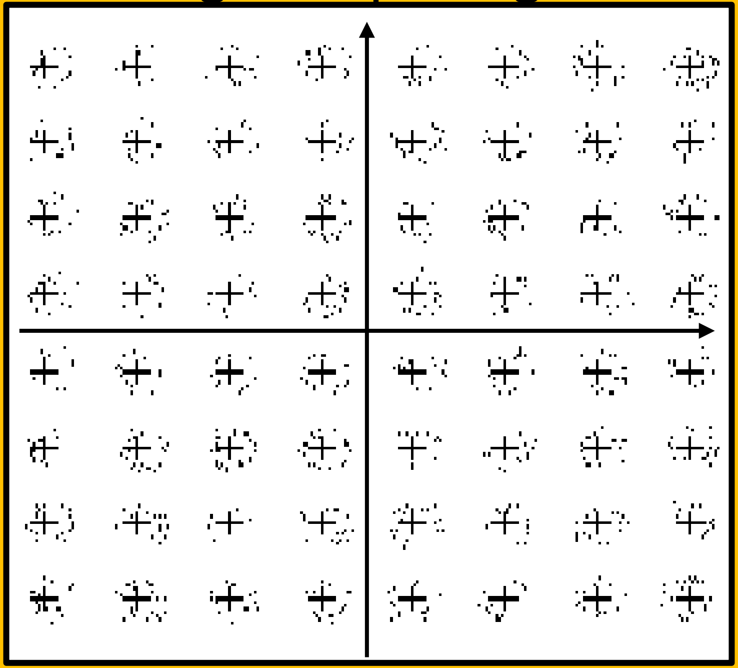


Distortion 6: Additive close-to-carrier spur signal (continued)



Distortion 6: Additive close-to-carrier spur signal

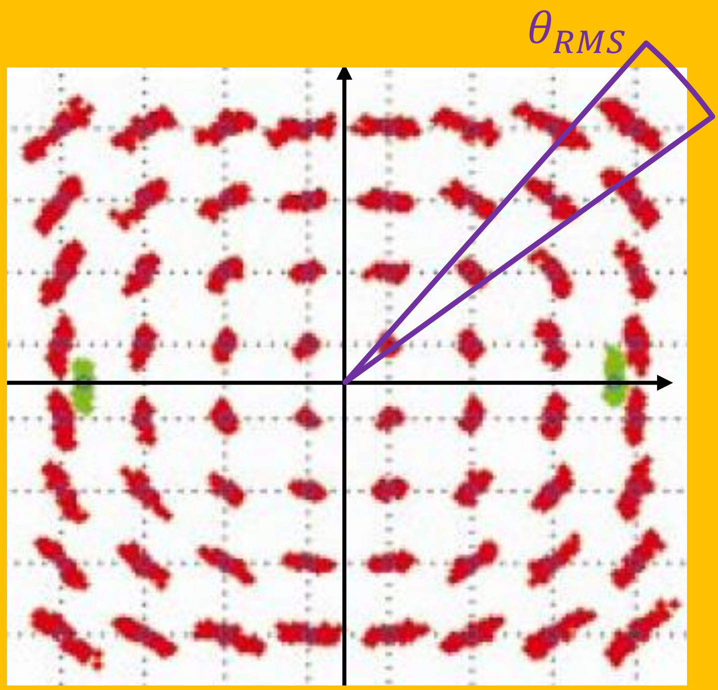
The understanding of this distortion is based upon the understanding of the previous one (carrier leakage). **However**, this time, it is not the carrier itself which is leaking into the spectrum of the modulated RF signal; It is a close-to-carrier (frequency) spur (interference) signal. Assuming the spur signal is located at fixed frequency offset from the carrier, its phasor representation with respect to the I axis will rotate at a rate which equals the difference frequency (between the carrier to the spur). This distortion will appear on the VSA as "circular rings" of samples (vectors) surrounding the standard constellation points.



VSA measurement of a Constellation with a close spur

☑ Systematic error

Distortion 7 Phase noise "smearing"



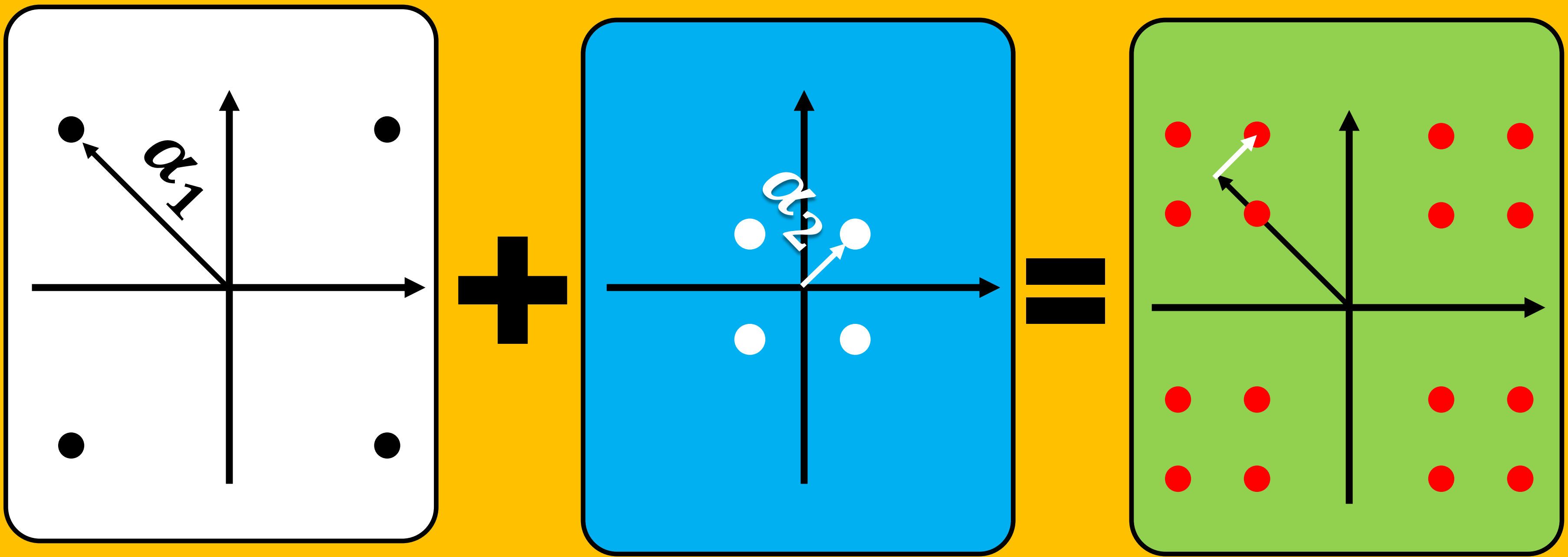
In case the I and Q carriers suffer from significant phase noise, the constellation symbols will become "smeared" by "phase noise arches".

The length on each arch will be proportional to the symbol's voltage envelope (distance from origin) and to the RMS phase noise value of the carriers.

X **Not a systematic error**

Distortion 8 Inter-Symbol Interference (ISI)

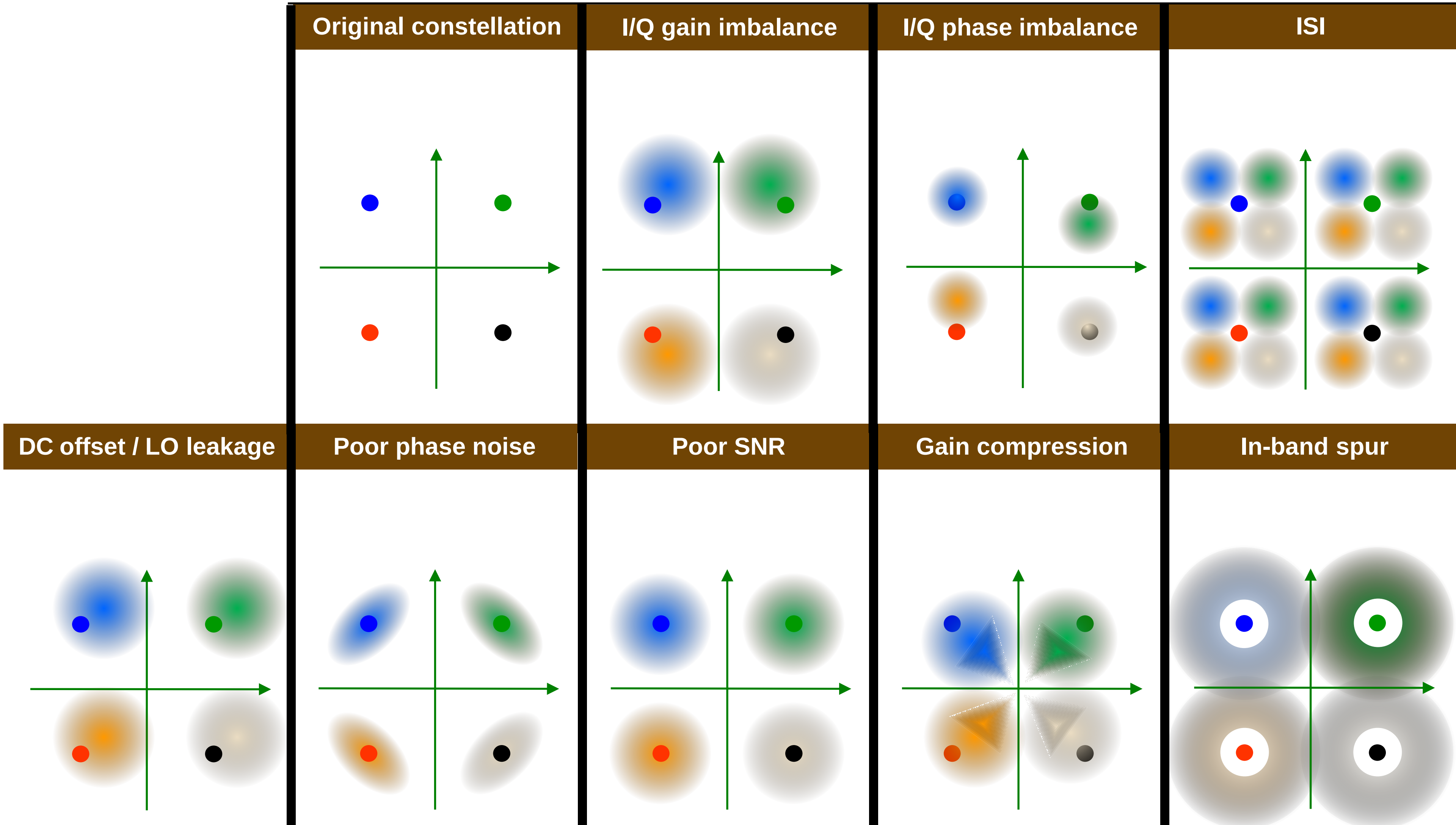
In case a constellation passes through a static multi-path channel (a memory device), the receiver will receive a superposition of the current symbol with past symbols, causing ISI:



☑ Systematic error

16.1 Distinct signal distortion mechanisms in the signal space

In VSA measurements, users can identify up to 8 distinct forms of signal distortions, each caused by a specific HW bottleneck:

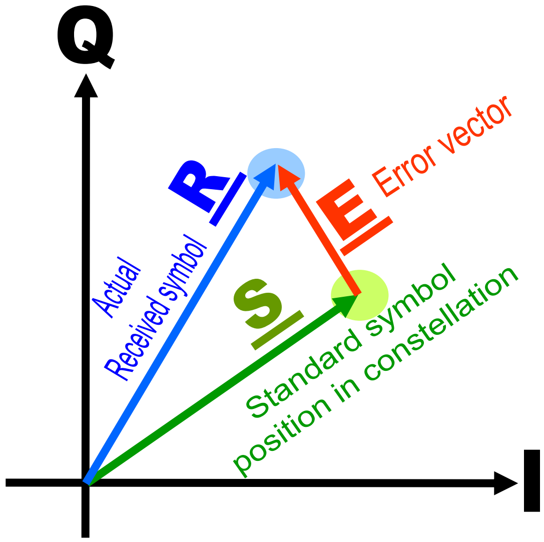


16.2 Definition of EVM

EVM is a “bottom line” metric of the purity of a digitally modulated signal (either transmitted or received).

It represents the difference (ratio) between the measured, distorted constellation to the ideal constellation.

Per symbol, EVM is defined as the ratio is between the RMS error vector magnitude to the desired signal. Hence, the smaller the EVM the better the signal quality.



$$EVM = E/S$$

EVM definition for a single symbol

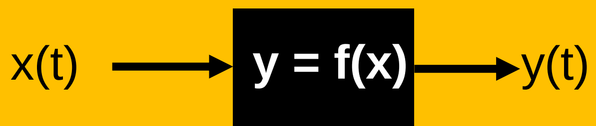
16.3 Baseband representation of signals and SISO analysis

The static multipath wireless channel

The static (zero doppler) multipath channel, is an LTI (Linear Time Invariant) system with a real-valued time-domain impulse response, shaped as an impulse train of different magnitudes and delays:

LTI systems - Reminder:

A system y = f(x) is an LTI system if both the linearity and time-invariance conditions are satisfied:



Linearity:

For every $\alpha_1, \alpha_2 \in \mathbb{R}$:

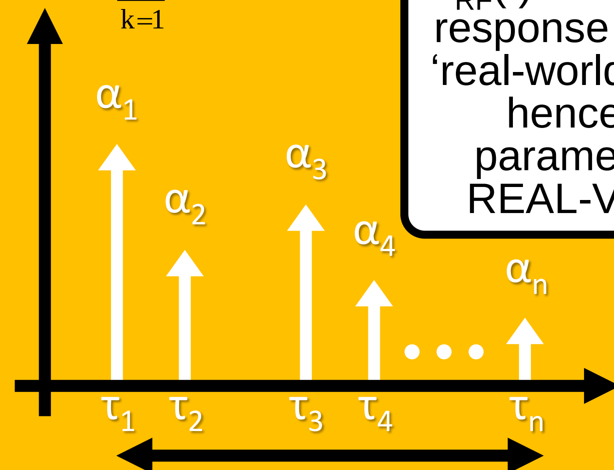
$$f[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = \alpha_1 f[x_1(t)] + \alpha_2 f[x_2(t)]$$

Time-invariance:

If the output due to an input $x(t)$ is $y(t)$, then the output due to a time-shifted input $x(t-t_0)$ is $y(t-t_0)$

The impulse response of the physical (hence real-valued) RF channel.

$$h_{RF}(t) = \sum_{k=1}^{n} \alpha_k \delta(t - \tau_k)$$



Pay attention:

$h_{RF}(t)$ is the impulse response of the RF, 'real-world' channel, hence all its parameters are REAL-VALUED.

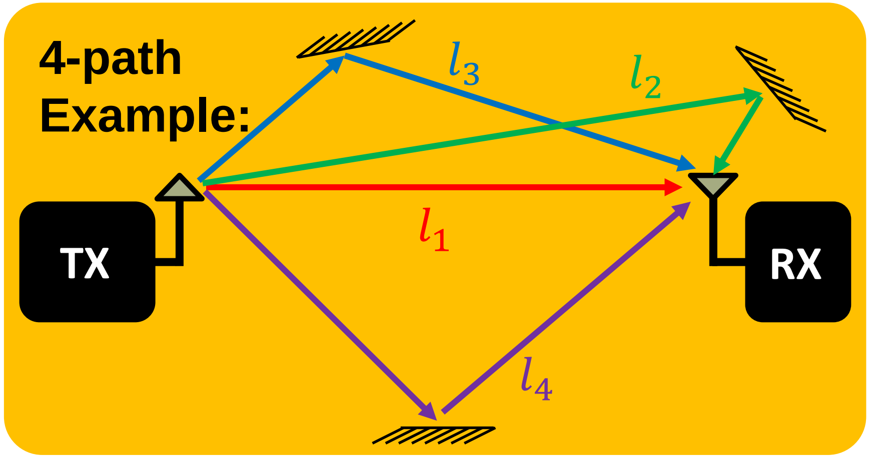
Delay Spread (DS) =

$$\max(\tau_i) - \min(\tau_i)$$

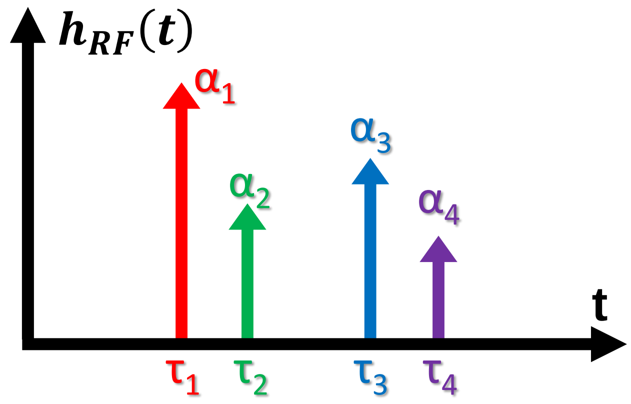
$h_{RF}(t) \in R$, $\{\alpha_k\}_{k=0}^{n} \in R$, $\{\tau_k\}_{k=0}^{n} \in R$

The static multipath wireless channel

As an example, consider the following static multipath channel with 4 paths (also known as "taps"). Its time-domain impulse response is real-valued and is in the from of a series of delta-functions, each scaled according to its specific path gain / loss, and time shifted according to its specific path's delay:



$$h_{RF}(t) = \alpha_1\delta\left(t-\underbrace{\frac{l_1}{c}}_{\tau_1}\right)+\alpha_2\delta\left(t-\underbrace{\frac{l_2}{c}}_{\tau_2}\right)+\alpha_3\delta\left(t-\underbrace{\frac{l_3}{c}}_{\tau_3}\right)+\alpha_4\delta\left(t-\underbrace{\frac{l_4}{c}}_{\tau_4}\right)$$



$$\boxed{h_{RF}(t)\in R},\ \boxed{\{\alpha_k\}_{k=0}^{n}\in R},\ \boxed{\{\tau_k\}_{k=0}^{n}\in R}$$

Complex Base-Band representation of real-valued signals:

Consider the following real-valued, general modulated RF signal:

$$r(t) = \underbrace{A(t)}_{env.} \cdot \cos[\omega_c t + \underbrace{\emptyset(t)}_{phase}] \in \mathbb{R}$$

Its two information-bearing entities are its **amplitude (envelope),** $A(t)$ and **Phase,** $\emptyset(t)$**.**

The Base-Band representation of r(t), namely s(t), represents these information-bearing entities in phasor representation, with the carrier wave being the reference phasor.

$$s(t) = A(t)e^{j\emptyset(t)} \in \mathbb{C}$$

The relationship between the real-valued RF signal r(t) and its BB representation **s**(t) is given by:

$$r(t) = \mathrm{Re}\{s(t)e^{j\omega_c t}\} =$$

$$= \mathrm{Re}\left\{\underbrace{A(t)e^{j\emptyset(t)}}_{s(t)} e^{j\omega_c t}\right\} = \mathrm{Re}\{A(t)e^{j[\omega_c t + \emptyset(t)]}\}$$

System related notations (noiseless, static LTI channel):

Transmitter:

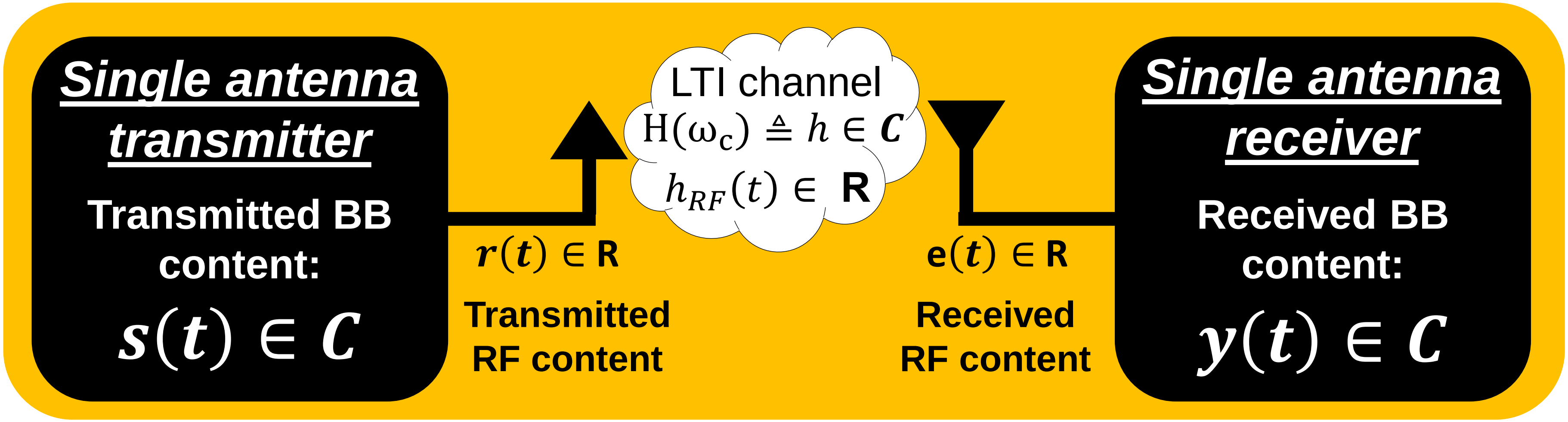
- The transmitted real-valued RF signal: $r(t) \in \mathbf{R}$
- The transmitted complex-valued BB signal: $s(t) \in \boldsymbol{C}$

Channel:

- The RF channel's real-valued, time-domain impulse response: $h_{RF}(t) \in \mathbf{R}$
- The RF channel's complex frequency response, evaluated at ω_c: $\mathrm{H}(\omega_c) \triangleq \mathbf{h} \in \boldsymbol{C}$

Receiver:

- The received real-valued RF signal: $e(t) \in \boldsymbol{R}$
- The received complex-valued BB signal: $y(t) \in \boldsymbol{C}$



The static (noiseless) LTI multipath wireless channel

We will now show that in steady-state, for an input sinusoid (CW tone), an LTI system can only modify the gain and / or phase of its input CW tone:

Consider the transmitted (real) CW RF signal:

$$r(t) = A\cos(\omega_c t + \emptyset) = Re\left\{ \underbrace{\underbrace{Ae^{j\emptyset}}_{s(t),TX} e^{j\omega_c t}}_{BB\ Phasor} \right\}$$

Transmitted RF

The received (real) RF signal is the transmitted (real) RF signal, **convolved** with the (also real) channel's impulse response:

Received RF

$$e(t) = \int_{-\infty}^{\infty} \boldsymbol{r(t-\tau)}\, h_{RF}(\tau) d\tau$$

And by substituting r(t) we get:

$$e(t) = \int_{-\infty}^{\infty} \underbrace{Re\left\{ \underbrace{\underbrace{Ae^{j\emptyset}}_{s(t),TX} e^{j\omega_c (t-\tau)}}_{BB\ Phasor} \right\}}_{\boldsymbol{r(t-\tau)}} h_{RF}(\tau) d\tau$$

Received RF

Which algebraically equals:

$$\underbrace{e(t)}_{Received\ RF} = Re\left\{ \int_{-\infty}^{\infty} \underbrace{\underbrace{Ae^{j\emptyset}}_{s(t),TX}}_{BB\ Phasor} e^{j\omega_c (t-\tau)} h_{RF}(\tau) d\tau \right\}$$

The static (noiseless) LTI multipath wireless channel

We will now show that in steady-state, for an input sinusoid (CW tone), an LTI system can only modify the gain and / or phase of its input CW tone:

Which also algebraically equals:

$$\underbrace{e(t)}_{\text{Received RF}} = Re\left\{ \underbrace{Ae^{j\phi}}_{\substack{s(t)\\ transmitted\\ Phasor}} \underbrace{\left(\int_{-\infty}^{\infty} e^{-j\omega_c\tau} h_{RF}(\tau) d\tau \right)}_{\substack{Fourier\ Transform\ of\ h_{RF}(t)\\ evaluated\ at\ \omega_c:\\ H(\omega_c) \triangleq \mathbf{h}}} e^{j\omega_c t} \right\}$$

Denoting $\mathbf{h} \triangleq H(\omega_c) \in \mathbb{C}$, to be the frequency response (transfer function) of the channel, at frequency ω_c, we get:

$$\underbrace{e(t)}_{\text{Received RF}} = Re\left\{ \underbrace{Ae^{j\phi}\mathbf{h}}_{\substack{y(t),RX\\ BB\ Phasor}} e^{j\omega_c t} \right\} = Re\left\{ y(t)\, e^{j\omega_c t} \right\}$$

The static (noiseless) LTI multipath wireless channel

To summarize, in steady state:

The channel can only change the magnitude and phase of the input CW. In BB representation (for a noiseless channel), the received BB signal, y(t) is given by the transmitted BB signal, s(t), multiplied by the channel's transfer function at ω_c, h:

BB (complex) notation:

$s(t) = A(t)e^{j\emptyset(t)}$

LTI channel

$H(\omega_c) \triangleq h \in \mathbb{C}$

$h_{RF}(t) \in \mathbb{R}$

$y(t) = s(t)h$

RF notation:

$r(t) = A\cos(\omega_c t + \emptyset)$

$e(t) = A|H(\omega_c)|\cos[\omega_c t + \emptyset + \angle H(\omega_c)]$

Amplitude change ($|H(\omega_c)|$)

Phase change ($\angle H(\omega_c)$)

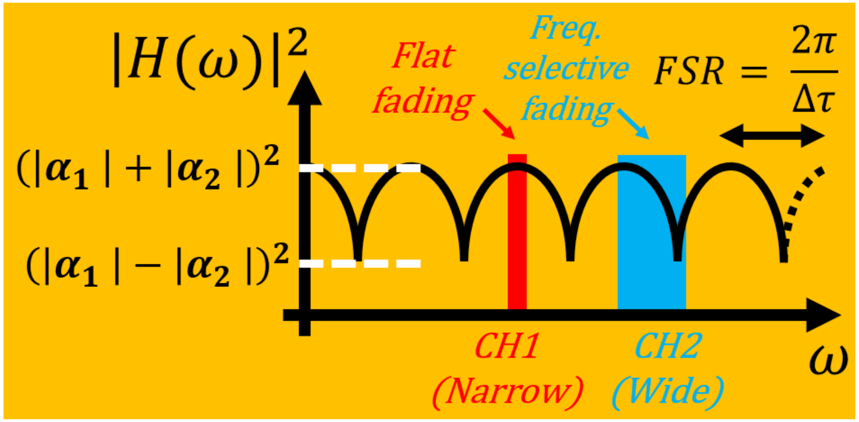
Flat fading vs. frequency selective fading

A “Flat” channel is a channel that passes all spectral components of the transmitted signal with approximately equal gain and linear phase over frequency.

A **true** flat-fading channel may include only one path (single delta function in the time domain impulse response), otherwise the channel will not be frequency flat due to the vector summation of complex exponents in the frequency domain.

However, when $DS \ll T_{sym}$, a flat fading channel model may practically be applied (such as in the case of OFDM sub-carriers).

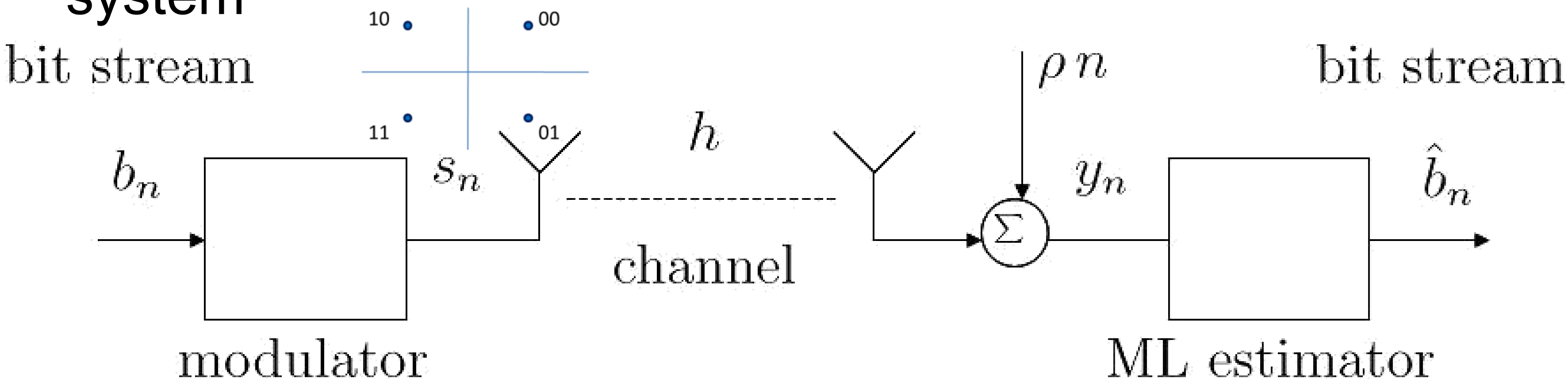
In practice, to apply a flat-fading model, we will consider **h** as constant over the signal’s BW.



BER vs. SNR analysis in AWGN SISO Channels

The SISO model

- We begin with the simplest digital SISO communications system

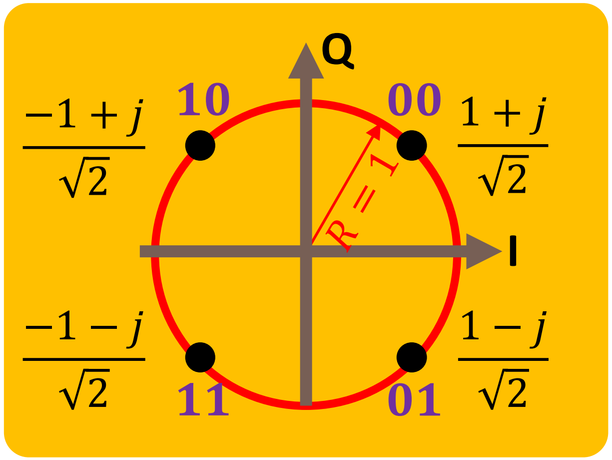


Here h is a flat fading complex channel (does not change in frequency). The noise n is standard Complex Normal

- The symbol s is drawn from QPSK modulation with unit power.

The SISO model - basic assumptions

Assumption 1:
The transmitted signal is a **normalized** (average power = $1v^2$) QPSK constellation:



Assumption 2:
The noise in the receiver is a standard complex-normal RV with zero mean and unity variance, multiplied by noise intensity ρ^2.

Assumption 3:
h is a flat fading LTI channel; hence **h** is hereby considered a single complex number (complex scalar).

Assumption 4:
The receiver knows the transfer function of the channel, **h** (i.e. channel estimation is applied).

The SISO model - noise

In our model, we consider the noise to be a Standard Complex Normal process, with noise intensity ρ^2.

A few words about circularly symmetric complex Normal distribution:

Consider two real-valued, i.i.d Gaussian variables, x and y, with zero mean and variance σ^2: $\boldsymbol{x \sim N(0, \sigma^2), y \sim N(0, \sigma^2)}$

Their joint pdf is:

$$pdf(x, y) = \frac{1}{2\pi\sigma^2} exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

We will now define a complex valued random variable $z = x + jy$.

Note that the moments of z are: $E\{z\} = 0$ and $E\{z^2\} = 2\sigma^2 = \sigma_z^2$

A **standard** complex-normal variable, z will have unity variance, i.e.:

$$\boldsymbol{z \sim CN(0, 1) \Longrightarrow x \sim N(0, 0.5), y \sim N(0, 0.5)}$$

Thus, in our case, the noise power will be ρ^2

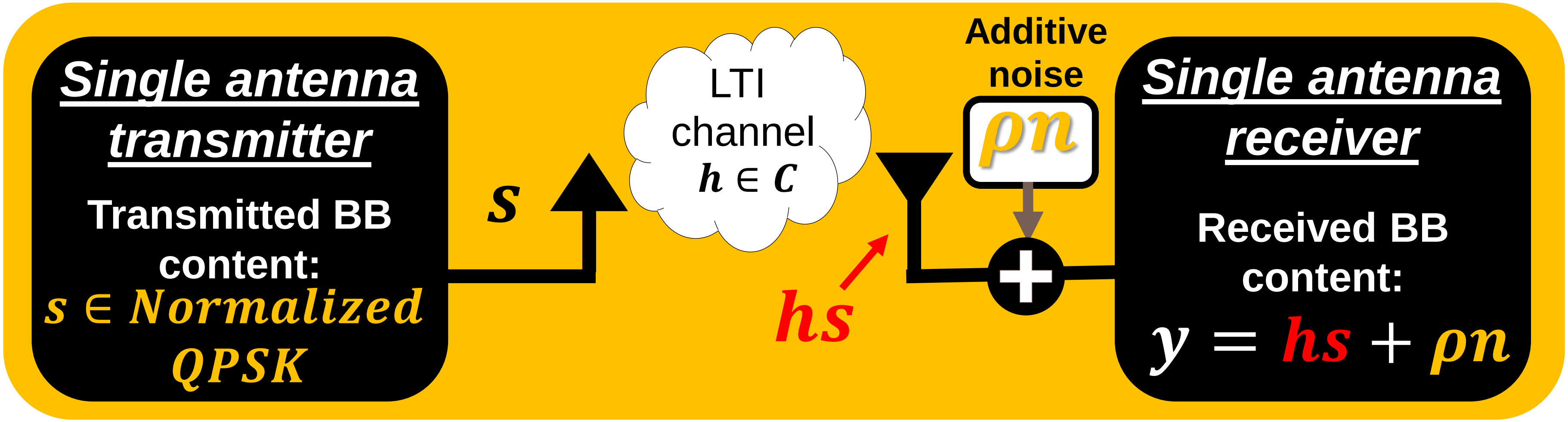
BER vs. SNR in SISO AWGN channels (1)

An AWGN channel means that the complex scalar **h** which represents the BB channel, remains constant over time (same **h** value for all the transmitted symbols).

In this case, the BB model for the received signal is as simple as:

$$y = \underbrace{hs}_{\text{Signal part}} + \underbrace{\rho n}_{\text{Noise part}}$$

Received BB



In our model, the instantaneous SNR (also the average SNR as **h** is fixed) is:

$$\underbrace{y}_{\text{Received BB}} = \underbrace{hs}_{\text{Signal part}} + \underbrace{\rho n}_{\text{Noise part}} \rightarrow SNR_{\text{AWGN, SISO}} = \frac{E\{(hs)^2\}}{E\{(\rho n)^2\}} = \frac{|h|^2}{\rho^2}$$

What would be a good receiver?

The role of the receiver would be to detect s (the transmitted QPSK symbol), **given** the measurement, y, assuming **h** is known.

An optimal receiver would detect s with minimum error probability ➔ MAP detector.

Reminder: MAP criteria basically says:

"Given y, the received (measured at the receiver) BB signal, what legal s (BB TX symbol) is the most probable that was transmitted?

➔ What symbol s is the most probable, if we know the given y measurement.

$$\widetilde{s_{MAP}} = argmax_{s \in QPSK} Prob(s / y)$$

$$\widetilde{s_{MAP}} = argmax_{s \in QPSK} Prob(s / y)$$

Bayes formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\widetilde{s_{ML}} = argmax_{s \in QPSK} \frac{\overbrace{Prob(y / s)Prob(s)}^{prob(y \cap s)}}{Prob(y)}$$

Reminder: Maximum Likelihood (ML) criteria basically says:

"What symbol s, makes the received BB signal y, most probable?

➔ **ML chooses the symbol that makes the measurement most probable.**

$$\widetilde{s_{ML}} = argmax_{s\in QPSK} \frac{Prob(y/s)Prob(s)}{Prob(y)}$$

0.25

Does not depend on s

$$\widetilde{s_{ML}} = argmax_{s\in QPSK} Prob(y/s)$$

Due to the normal distribution of the noise, the pdf of y/hs is: $(y/h\,;s) \sim CN(hs,\rho^2)$

Selected constellation point

$$\tilde{s} = \arg\max_{s\in \mathrm{QAM}} \frac{1}{\pi\rho^2}\exp\left(-\frac{|y-hs|^2}{\rho^2}\right) = \arg\min_{s\in \mathrm{QAM}} |y-hs|^2$$

$$= \arg\min_{s\in \mathrm{QAM}} |\hat{s}-s|^2 ; \quad \hat{s} = \frac{y}{h}.$$

Equalized received symbol

This means that we first compensate for the effect of the channel and create $\hat{s} = y/h$, then we choose the point $\tilde{s}$ closet to $\hat{s}$.

We have just seen, that the optimal (ML) receiver will select the symbol:

$$\tilde{S}_{ML} = argmin_{s \in QAM} \{|\hat{s} - s|^2\}$$

Where:

The equalized (scaled & rotated) received symbol: $\hat{s} = \frac{y}{h}$

Standard QPSK constellation point

<u>This means:</u>

select the STANDARD constellation point, $\tilde{S}_{ML}$, that is CLOSEST to the equalized received symbol, $\hat{S}$.

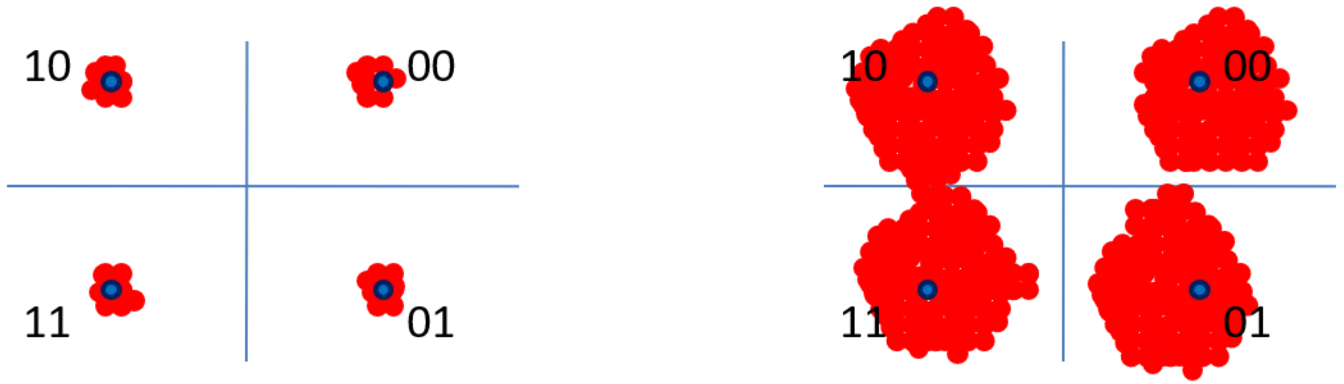
Note that the equalized constellation point was actually offset by the noise from its standard place in the constellation. In other words:

The channel compensated point $\hat{S}$ is made up of the original transmitted symbol s and a noisy term:

$$\hat{s} = \frac{y}{h} = \frac{hs + \rho n}{h} = s + \frac{\rho n}{h}$$

Noisy part

In high SNR, the intensity of the noisy term is small and $\hat{s}$ is distributed around the original constellation points.

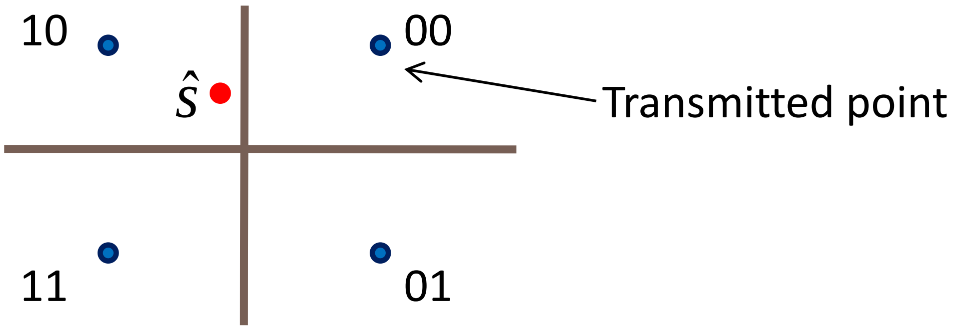


High SNR | Low SNR

In SISO, the instantaneous SNR in $\hat{s}$ is the original SNR:

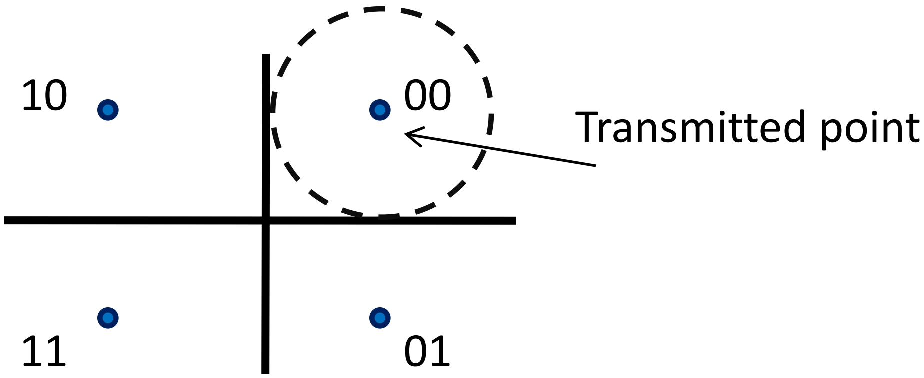
$$\frac{|h|^2}{\rho^2}$$

Occasionally, the noisy term throws $\hat{s}$ out of the right decision region and an error occurs.



What is the (upper boundary for the) error probability?

- Of course we can find the error probability with Q functions…
- But we want a **simple expression** – so we will look at the probability to step out of a circle with radius $d_{\min}/2$



- So we want to compute the upper bound

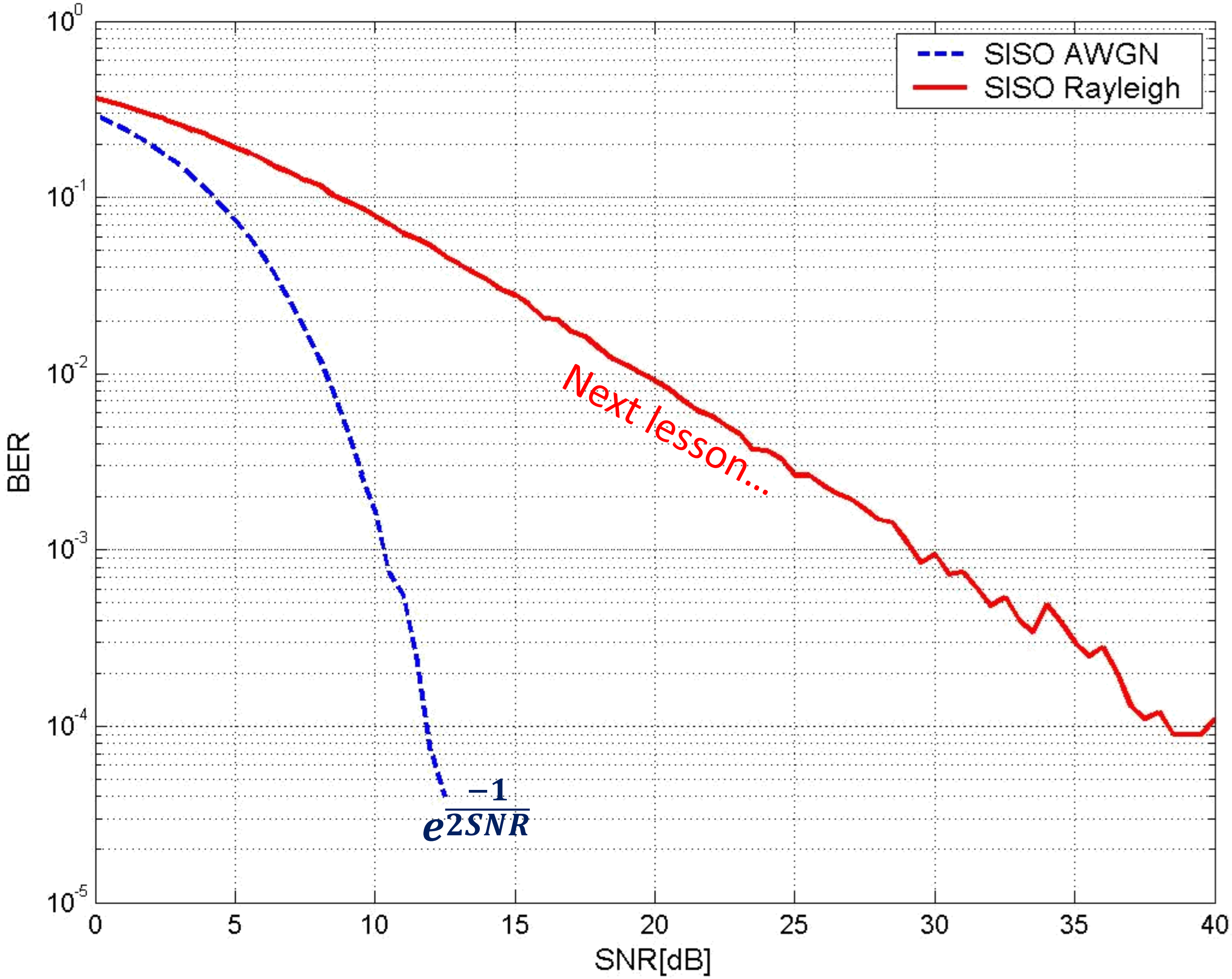
$$\Pr\{\text{error} \mid h\} \le \Pr\left\{\left|\frac{\rho n}{h}\right| > \frac{1}{\sqrt{2}} \mid h\right\} = \Pr\left\{|n| > \frac{|h|}{\sqrt{2}\,\rho} \mid h\right\}$$

- We use the fact that $z=|n|$ is Rayleigh distributed with so we have

$$\Pr\{\text{error}\,|\,h\}\leq\int_{z=\frac{|h|}{\sqrt{2}\rho}}^{\infty}2z\,e^{-z^2}dz=\exp\left(-\frac{|h|^2}{2\rho^2}\right)=\exp\left(-\frac{\text{SNR}(h)}{2}\right)$$

<u>**Note**</u>: The absolute value z of a complex Normal RV $x+iy$ where x and y are zero mean real valued iid Gaussian RVs each with variance σ^2 is Rayleigh distributed with parameter σ and pdf

$$p(z)=\frac{1}{\sigma^2}z\exp\left(-\frac{z^2}{2\sigma^2}\right)\quad\text{for}\quad z>0$$



SISO AWGN
SISO Rayleigh
BER
SNR[dB]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
5
10
15
20
25
30
35
40
Next lesson...
$e^{\frac{-1}{2SNR}}$

Any questions?

That's it for today!
Don't forget today's homework!

Thank you for attending and see you next week!

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